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A new Active Control method for Chaos Synchronization

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Abstract— **A new active control method is proposed for two different chaotic systems synchronization. Based on compound matrices, this method is compared with the generalized active control one through a numerical example. The Shimizu-Morioka and Pan chaotic systems are considered in order to show the advantages of the proposed method.**

Index Terms— **chaos, synchronization, active control, compound matrices.**

I. Introduction

Chaos synchronization is an attractive phenomenon in science and technology that involves a variety of real-life processes. Secure telecommunication, biology, chemistry and medicine are some fields where chaos synchronization occurs or can be exploited.

Chaotic systems are nonlinear systems distinguished by their high sensitivity to initial conditions variation. In 1990, Pecora and Carrol proved [1] that two chaotic systems can synchronize. This means that a first system (response system), can follow the trajectories of a second one (drive system), when a suitable control law is applied.

Many synchronization schemes have been proposed [2, 3, 4, 5] such as nonlinear control [6], nonlinear observer [4, 7, 8], adaptive control [9], active control [10, 11, 12], fuzzy control [13, 14], and backstepping control [15, 16]…

In this paper, we propose an active control scheme based on the concept of compound matrices, in order to synchronize two different chaotic systems.

Compound matrices [17, 18], due to their interesting spectral properties, are a powerful tool for stability study [17, 19]. In [18], stability of matrices and existence of Hopf bifurcation in dynamical systems analysis are investigated using the compound matrices formalism.

The method proposed in this work, which is a variant of active control, is compared to the generalized active control technique. For this purpose, we apply both techniques to a concrete example: synchronization of the Shimizu-Morioka and Pan chaotic systems. Numerical simulations are performed to illustrate the effectiveness of the proposed method and its advantages.

In the second section we present the problem statement and introduce the mathematical background of the proposed method which will be detailed in Section III. The problem of synchronizing the Shimizu-Morioka and Pan chaotic systems is considered in Section IV, for an illustrative purpose. Obtained results, are compared to those obtained by the generalized active method.

II. Preliminaries

Given two chaotic systems, as described by (1) and (2) corresponding respectively to a drive and a response system, our aim is to design a control law U such that these two systems synchronize.

$$\dot{X} = f(X) \tag{1}$$

$$\dot{Y} = g(Y) + U \tag{2}$$

$X = (x_1,...,x_n)$ and $Y = (y_1,...,y_n)$ are the state vectors of the two systems and $U = (u_1,...,u_n)$ is the control vector depending on X and Y to be calculated.

Let $e = (e_1,...,e_n) = (y_1 - x_1,..., y_n - x_n)$ be the synchronization error vector. Then synchronization is achieved when the error dynamical system (3) is stable.

$$\dot{e} = h(e, X, Y) \tag{3}$$

Our proposed approach is based on the compound matrix method. So, we give a brief overview, introduced in [18, 19], about this mathematical concept.

Let $M_n(\mathrm{R})$ be the linear space of matrices of size n x n with entries in R and let A be a matrix in $M_n(\mathrm{R})$ and k an integer in $[1,n]$. We note by $\wedge$ the exterior product in R^n.

Definition 1 [17, 18] The additive compound matrix $A^{[k]}$ of A, with respect to the canonical basis in the k^{th} exterior product space $\wedge^k \mathrm{R}^n$ is a linear operator on $\wedge^k \mathrm{R}^n$ and can be defined on a decomposable element $u_1 \wedge u_2 \wedge ... \wedge u_k$ by

$$A^{[k]}(u_1 \wedge ... \wedge u_k) = \sum_{i=1}^{k} u_1 \wedge ... \wedge Au_i \wedge ... \wedge u_k \ , \quad \forall u_1 \wedge ... \wedge u_k \in \mathrm{R}^n \tag{4}$$

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Relations between entries (a_{ij}) of A and those of $A^{[k]}$ ($\tilde{a}_{ij}$) are linear.
Let i be an integer in $[1, C_n^k]$. If we note by $(i) = (i_1,...,i_k)$ the i^{th} member in the lexicographic ordering of integer k-tuples such that $1 \le i_1 < ... < i_k \le n$, we can obtain the additive compound matrix entries from the following result.

Proposition 1 [17, 18]

$$\tilde{a}_{ij} = \begin{cases} a_{i_1 i_1} + ... + a_{i_k i_k}, & if\ (i) = (j), \\ (-1)^{r+s} a_{j_r i_s}, & if\ exactly\ one\ entry\ i_s\ of\ (i)\ does\ not\ occur\ in\ (j) \\ & and\ j_r\ does\ not\ occur\ in\ (i), \\ 0 & if\ (i)\ differs\ from\ (j)\ in\ two\ or\ more\ entries. \end{cases} \quad (5)$$

In particular, we have $A^{[1]} = A$, $A^{[n]} = trace(A)$ and for $A \in M_3(\mathbb{R})$

$$A^{[2]} = \begin{pmatrix} a_{11} + a_{22} & a_{23} & -a_{13} \\ a_{32} & a_{11} + a_{33} & a_{12} \\ -a_{31} & a_{21} & a_{22} + a_{33} \end{pmatrix} \quad (6)$$

Definition 2 [18] Let $|.|$ a vector norm on $M_n(\mathbb{R})$ and A a matrix in $M_n(\mathbb{R})$.
The Lozinskiĭ measure (logarithmic measure) μ of A with respect to $|.|$ is defined by

$$\mu(A) = \lim_{h \to 0^+} \frac{|I + hA| - 1}{h} \quad (7)$$

As examples, Lozinskiĭ measure of a matrix A with respect to the three common vector norms
$|x|_1 = \sum_i |x_i|$, $|x|_2 = \sqrt{\sum_i |x_i|^2}$ and $|x|_\infty = \sup_i |x_i|$ are

$$\mu_1(A) = \sup_j (a_{jj} + \sum_{i, i \ne j} |a_{ij}|),$$
$$\mu_2(A) = s(\frac{A + A^T}{2}) \quad (8)$$
and $\mu_\infty(A) = \sup_i (a_{ii} + \sum_{j, j \ne i} |a_{ij}|)$

where $s(A)$ denotes the maximum real part of the eigenvalues of A.

Due to their interesting spectral properties, compound matrices present a powerful tool for the stability study of matrices.
We present, in the following, an important result about compound matrices that will be used in the sequel.

Theorem 1 [18] If $(-1)^n \det(A) > 0$ then A is stable if and only if there exists a Lozinskiĭ measure μ on $M_m(\mathbb{R})$ such that $\mu(A^{[2]}) < 0$, $m = C_n^2$.

III. Proposed active control

Given the state vectors $X = (x_1,...,x_n)$ for the drive system and $Y = (y_1,...,y_n)$ for the response system, we define the extended state vector φ such as

$$\varphi = (x_1,...,x_n, y_1,...,y_n) \quad (9)$$

The synchronization error becomes

$$e = T\varphi \quad (10)$$

with $T = (-I_n, I_n) \in M_{n,2n}(\mathbb{R})$; I_n denotes the identity matrix in $M_n(\mathbb{R})$.
We suppose that the error dynamical system (3) can be expressed as

$$\dot{e} = N(\varphi)\varphi + U \quad (11)$$

Our purpose is to find a state feedback law $U = -K(\varphi).\varphi$, $K(\varphi) \in M_{n,2n}(\mathbb{R})$, stabilizing the system

$$\dot{e} = (N(\varphi) - K(\varphi)\varphi \quad (12)$$

If we look for a special choice of the matrix $K(\varphi)$ such that

$$N(\varphi) - K(\varphi) = A(K, \varphi)T \quad (13)$$

Error system description (12) becomes in the form:

$$\dot{e} = A(K, \varphi)T\varphi \quad (14)$$

where $A(K, \varphi) \in M_n(\mathbb{R})$ is a matrix to be expressed in terms of the gain matrix entries.
By substituting (10) in (14) we obtain the new formulation of the dynamical error system

$$\dot{e} = A(K, \varphi)e \quad (15)$$

Remark: Given the particular structure of matrix T and partitioning $N(\varphi)$ and $K(\varphi)$ into two n x n matrices ($N_1(\varphi)$, $N_2(\varphi)$) and ($K_1(\varphi)$, $K_2(\varphi)$), equation (13) reduces to

$$N_1(\varphi) - K_1(\varphi) = -A(K, \varphi) \quad (16)$$

Then, the second gain matrix block $K_2(\varphi)$ will be deduced from the relation

$$N_2(\varphi) - K_2(\varphi) = A(K, \varphi) \quad (17)$$

Using the description detailed above for the error dynamical system, the synchronization of the two chaotic systems (1) and (2) consists of stabilizing the system (15).
For this aim, we propose the following results.

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Theorem 2 If there exist two matrices $K_1(\varphi)$ and $K_2(\varphi) \in M_n(\mathbb{R})$, with $K_1(\varphi) - N_1(\varphi) = N_2(\varphi) - K_2(\varphi)$ and such that the matrix $A(K,\varphi) = K_1(\varphi) - N_1(\varphi)$ fulfils the two following conditions:

(i) $(-1)^n \det(A(K,\varphi)) > 0$

(ii) $\mu(A^{[2]}(K,\varphi)) < 0$ for some Lozinskii measure μ on $M_m(\mathbb{R})$, $m = C_n^2$

then, global synchronization is achieved between the drive system (1) and the response system (2) by applying the control law

$$U = -K_1(\varphi)X - K_2(\varphi)Y \tag{18}$$

Proof Immediate by applying theorem 1 to study the stability of the error dynamical system (15).

Corollary 1 Global synchronization between the chaotic systems described by (1) and (2), with respect to the error dynamical system description (15), is achieved by applying the control law

$$U = -K_1(\varphi)X - K_2(\varphi)Y \tag{19}$$

if there exist two matrices $K_1(\varphi)$ and $K_2(\varphi) \in M_n(\mathbb{R})$ with $K_1(\varphi) - N_1(\varphi) = N_2(\varphi) - K_2(\varphi)$ and such that the matrix $A(K,\varphi) = K_1(\varphi) - N_1(\varphi)$ fulfils the two following conditions:

(i) $(-1)^n \det(A(K,\varphi)) > 0$

(ii) $\forall j = 1..m$, $\tilde{a}_{jj} + \sum_{i=1..m,\, i \neq j} |\tilde{a}_{ij}| < 0$ with $\tilde{a}_{ij}$ being the entries of $A^{[2]}(K,\varphi)$ and $m = C_n^2$.

Proof A direct application of theorem 2 by considering the Lozinskiĭ measure of $A^{[2]}(K,\varphi) = (\tilde{a}_{ij})$ with respect to the norm $|x|_1 = \sum_i x_i$: $\mu_1(\tilde{A}) = \sup_j (\tilde{a}_{jj} + \sum_{i, i \neq j} |\tilde{a}_{ij}|)$

It's clear that $\mu_1(\tilde{A}) < 0$ if for every column j of $\tilde{A}$, $\tilde{a}_{jj} + \sum_{i,\, i \neq j} |\tilde{a}_{ij}| < 0$.

Corollary 2 Global synchronization between the chaotic systems described by (1) and (2), with respect to the error dynamical system description (15), is achieved by applying the control law

$$U = -K_1(\varphi)X - K_2(\varphi)Y \tag{20}$$

if there exist two matrices $K_1(\varphi)$ and $K_2(\varphi) \in M_n(\mathbb{R})$ with $K_1(\varphi) - N_1(\varphi) = N_2(\varphi) - K_2(\varphi)$ and such that the matrix $A(K,\varphi) = K_1(\varphi) - N_1(\varphi)$ fulfils the two following conditions:

(i) $(-1)^n \det(A(K,\varphi)) > 0$

(ii) $\forall i = 1..m$, $\tilde{a}_{ii} + \sum_{j=1..m,\, j \neq i} |\tilde{a}_{ij}| < 0$ with $\tilde{a}_{ij}$ being the entries of $A^{[2]}(K,\varphi)$ and $m = C_n^2$.

Proof As in corollary 1 and by considering the Lozinskiĭ measure of $A^{[2]}(K,\varphi) = (\tilde{a}_{ij})$ with respect to the norm $|x|_\infty = \sup_i |x_i|$, it comes

$\mu_\infty(\tilde{A}) = \sup_i (\tilde{a}_{ii} + \sum_{j,\, j \neq i} |\tilde{a}_{ij}|)$

$\mu_\infty(\tilde{A}) < 0$ if for every row i of $\tilde{A}$, $\tilde{a}_{ii} + \sum_{j,\, j \neq i} |\tilde{a}_{ij}| < 0$.

In the next section, we give an application example for the use of the proposed method.

IV. Application to the Synchronization of Shimizu-Morioka and Pan Chaotic Systems

To test the proposed method, let's consider the synchronization between Shimizu-Morioka [21, 23] and Pan chaotic systems [22].

The Shimizu-Morioka system [21] is described by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1 - \lambda x_2 - x_1 x_3 \\ \dot{x}_3 = -\alpha x_3 + x_1^2 \end{cases} \tag{21}$$

with x_1, x_2 and x_3 the state variables of the system and λ and α its parameters. The Shimizu-Morioka system is chaotic when we have $\lambda = 0.605$ and $\alpha = 0.549$.

The Pan chaotic system [22], considered here as a response system, is ruled by

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + u_1 \\ \dot{y}_2 = c y_1 - y_1 y_3 + u_2 \\ \dot{y}_3 = y_1 y_2 - b y_3 + u_3 \end{cases} \tag{22}$$

y_1, y_2 and y_3 are the state variables of the system, a, b and c its parameters and u_1, u_2 and u_3 the control variables.

The autonomous Pan system is chaotic for $a = 10$ $b = \frac{8}{3}$ $c = 16$.

Defining the error sates by

$$e_1 = y_1 - x_1,\ e_2 = y_2 - x_2 \text{ and } e_3 = y_3 - x_3, \tag{23}$$

yields the synchronization error dynamical system

$$\begin{cases} \dot{e}_1 = a(y_2 - y_1) - x_2 + u_1 \\ \dot{e}_2 = c y_1 - y_1 y_3 - x_1 + \lambda x_2 + x_1 x_3 + u_2 \\ \dot{e}_3 = y_1 y_2 - b y_3 + \alpha x_3 - x_1^2 + u_3 \end{cases} \tag{24}$$

The error states vector and the error dynamical system can be written respectively under the form (10) and (12) with

$$T=\begin{pmatrix}-1&0&0&1&0&0\\0&-1&0&0&1&0\\0&0&-1&0&0&1\end{pmatrix} \text{ and } \quad (25)$$

$$N(\varphi)=\begin{pmatrix}0&-1&0&-a&a&0\\-1&\lambda&x_1&c&0&-y_1\\-x_1&0&\alpha&0&y_1&-b\end{pmatrix}$$

By solving the matrix equations (16) and (17) we obtain the expression of matrix $A(\varphi)$

$$A(\varphi)=\begin{pmatrix}k_{11}&1+k_{12}&k_{13}\\1+k_{21}&-\lambda+k_{22}&-x_1+k_{23}\\x_1+k_{31}&+k_{32}&-\alpha+k_{33}\end{pmatrix} \quad (26)$$

and the relations between the two blocks $K_1(\varphi)$ and $K_2(\varphi)$ of matrix $K(\varphi)$.

$$\begin{cases}k_{11}=-a-k_{14}\\1+k_{12}=a-k_{15}\\k_{13}=-k_{16}\end{cases}\quad\begin{cases}1+k_{21}=c-k_{24}\\-\lambda+k_{22}=-k_{25}\\-x_1+k_{23}=-y_1-k_{26}\end{cases}\quad\begin{cases}x_1+k_{31}=-k_{34}\\k_{32}=y_1-k_{35}\\-\alpha+k_{33}=-b-k_{36}\end{cases} \quad (27)$$

The expression of the compound matrix $A^{[2]}(\varphi)$ is deduced from $A(\varphi)$

$$A^{[2]}(\varphi)=\begin{pmatrix}k_{11}-\lambda+k_{22}&-x_1+k_{23}&-k_{13}\\k_{32}&k_{11}-\alpha+k_{33}&1+k_{12}\\-x_1-k_{31}&1+k_{21}&-\lambda+k_{22}-\alpha+k_{33}\end{pmatrix} \quad (28)$$

A possible choice of the gain matrix block $K_1(\varphi)$, with respect to the conditions of corollary 1, is the following

$$K_1(\varphi)=\begin{pmatrix}-|x_1|-\varepsilon&0&0\\0&0&0\\0&0&0\end{pmatrix} \quad (29)$$

where ε is a positive constant to be calculated.
This leads, according to the relations in (27), to the second block matrix $K_2(\varphi)$ given by

$$K_2(\varphi)=\begin{pmatrix}-a+|x_1|+\varepsilon&a-1&0\\c-1&\lambda&x_1-y_1\\-x_1&y_1&\alpha-b\end{pmatrix} \quad (30)$$

Under this choice, the new expression of the compound matrix $A^{[2]}(\varphi)$ and the determinant of the matrix $A(\varphi)$ are respectively

$$A^{[2]}(\varphi)=\begin{pmatrix}-|x_1|-\varepsilon-\lambda&-x_1&0\\0&-|x_1|-\varepsilon-\alpha&1\\-x_1&1&-\lambda-\alpha\end{pmatrix} \quad (31)$$

$$\det(A(\varphi))=-\lambda\alpha|x_1|-\lambda\alpha\varepsilon+\alpha-x_1^{2} \quad (32)$$

Corollary 1 conditions $\begin{cases}-\varepsilon-\lambda<0\\1-\varepsilon-\alpha<0\\-\lambda-\alpha<0\\-\lambda\alpha|x_1|-\lambda\alpha\varepsilon+\alpha-x<0\end{cases}$ are

equivalent to $\begin{cases}1-\lambda\varepsilon<0\\1-\varepsilon-\alpha<0\end{cases}$

which reduces, given the numerical values of λ and α, to $\varepsilon>\dfrac{1}{\lambda}=1.653$.

Finally, by fixing $\varepsilon=2$, a possible choice for the gain matrix $K(\varphi)$ is

$$K(\varphi)=\begin{pmatrix}-|x_1|-2&0&0&-a+|x_1|+2&a-1&0\\0&0&0&c-1&\lambda&x_1-y_1\\0&0&0&-x_1&y_1&\alpha-b\end{pmatrix} \quad (33)$$

Under this choice, the chaotic systems defined by (1) and (2) synchronize when applying the control law $U=-K(\varphi)\varphi$.
Numerical simulations, using the fourth order Runge-Kutta method with a time step size of 0.001, are given in Fig. 1.
The initial conditions considered for the drive and response systems are respectively (0.1, 0.2, 0.1) and (-0.5, 0.4, 0.5).

V. Comparison with Generalized Active Control

The Active Control method consists of calculating an appropriate state feedback controller U that stabilizes the error dynamical system (3).
In the Generalized Active Control we assume that the system (3) can be written, by separating the linear and the nonlinear parts, in the form

$$\dot e = A.e+\Phi(e,X,Y)+U \quad (34)$$

A is a constant matrix in $M_n(\mathbb{R})$ and the control law U is such that

$$U=-\Phi(e,X,Y)-K_g e \quad (35)$$

where $K_g\in M_n(\mathbb{R})$ is a linear gain matrix. Equation (34) becomes

$$\dot e=(A-K_g)e \quad (36)$$

Therefore, by calculating the convenient gain matrix K_g stabilizing the system (36), and substituting it in equation (35) we obtain the control law U leading to the synchronization between the drive and the response systems.

For comparative purpose, we consider the same example of Shimizu-Morioka and Pan chaotic systems ruled by equations (21) and (22).

The error dynamical system given by (24) can be expressed in the from (34), where

$$A=\begin{pmatrix}0 & a & 0\\ c & 0 & 0\\ 0 & 0 & -b\end{pmatrix} \text{ and } \Phi=\begin{pmatrix}-ay_1+(a-1)x_2\\ (c-1)x_1+\lambda x_2-y_1y_3+x_1x_3\\ (\alpha-b)x_3+y_1y_2-x_1^2\end{pmatrix}. \quad (37)$$

One possible choice for the gain matrix K_g for which the stability of the system (36) is guarantied is the following

$$K_g=\begin{pmatrix}16 & 0 & 0\\ 0 & 16 & 0\\ 0 & 0 & 0\end{pmatrix} \quad (38)$$

This choice imposes to the system (36) the eigenvalues -2.67, -3.35, -28.65, which implies its stability.
The resulting control law is given by (39) and simulation results are illustrated in Fig. 1 using the same simulation parameters as section 5.
In Table I. we recapitulate the control laws proposed for each method. We remark that in both cases three controllers are needed (u_1, u_2 and u_3) and all the three state variables of the response system are used.

TABLE I. COMPARISON OF THE CONTROL LAW EXPRESSIONS

Method	Control law expression
Generalized Active Control	$\begin{cases}u_1=16x_1-(a-1)x_2+(a-16)y_1\\ u_2=(1-c)x_1+(16-\lambda)x_2-x_1x_3-16y_2+y_1y_3\\ u_3=x_1^2-(\alpha-b)x_3-y_1y_2\end{cases}$ (39)
Proposed Active Control	$\begin{cases}u_1=(\lvert x_1\rvert+2)x_1+(a-\lvert x_1\rvert-2)y_1+(1-a)y_2\\ u_2=-x_1y_3+(1-c)y_1-\lambda y_2+y_1y_3\\ u_3=x_1y_1-y_1y_2+(b-\alpha)y_3\end{cases}$ (40)

Nevertheless, only one state variable of the drive system is used in our proposed control law (x_1), against 3 for the generalized active control (x_1, x_2 and x_3). This is a very important advantage in practical implementation.

According to the simulations results, it's clear that the generalized active control performs better, especially in terms of rapidity. But we assume that other choices for the proposed gain matrix can be made to enhance the synchronized system behaviour.

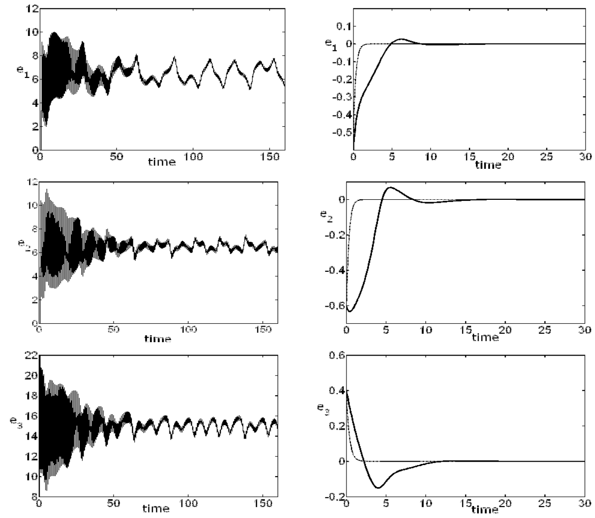

Fig. 1. Evolution of the synchronization error states of Example 1, when control is de-activated (on the left) and with control switched on at time = 0 (on the right)
(dashed line: generalized active control, solid line: proposed active control)

The complexity is similar in both controls but the design seems to be easier by the proposed method. On the one hand, the generalized active control requires usually an analytic development to express the relation between the synchronization error and its derivative. This can be done easily by programming in our method, using directly the extended state vector and exploiting the controller itself to reach the required expression form.
On the other hand, it is much easier to manipulate inequalities than calculating eigenvalues and this is another significant advantage for the proposed method.

VI. CONCLUSION

This paper proposed a new active control design method for chaotic systems synchronization. The method is based on compound matrices, and has been successfully applied to synchronize the Shimizu-Morioka and Pan chaotic systems. The simulation results are compared to those obtained by applying the generalized active technique. The proposed active control didn't perform better than the generalized active control in terms of rapidity, but presents some interesting advantage in practical implementation. Besides its flexibility and simplicity, the proposed method allows easier choice of the feedback gain matrix. This permits, inter alia, to reduce the drive state variables implied in the control law, which is a crucial issue in telecommunication application.
The proposed method can be enhanced to attain better performance, especially in terms of rapidity.

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